$\operatorname{div} \vec{B} = 0$ (structure mag.) $P : 0 \to \pm 0$

$P(\operatorname{div} \vec{B}) = -\operatorname{div} \vec{B} = 0$

$(\vec{\nabla} \cdot \vec{B})$

$\Leftrightarrow \operatorname{div} \vec{B} = 0$ ✓

$\vec{\operatorname{rot}}\, \vec{B} = \mu_0 \left(\vec{j} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \right)$ (Ampère)

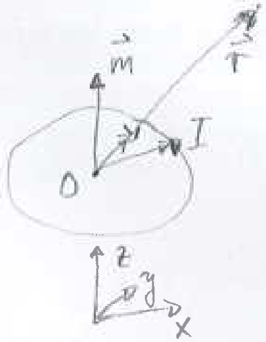
$P(\vec{\operatorname{rot}}\, \vec{B}) = -\vec{\operatorname{rot}}\, \vec{B}$

$P(\mu_0 \vec{j}) = -\mu_0 \vec{j}$ ✓

$P(\vec{E}) = -\vec{E}$

L'électromagnétisme conserve la parité spatiale.

e)



on suppose en sphérique que

$$\vec{m} = m \vec{e}_z = m(\cos\vartheta\, \vec{e}_r - \sin\vartheta\, \vec{e}_\vartheta)$$

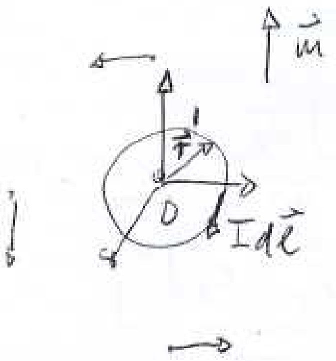
$$\vec{r} = r \vec{e}_r$$

$$\Rightarrow \vec{m} \times \vec{r} = m r (\cos\vartheta\, \vec{e}_r \times \vec{e}_r - \sin\vartheta\, \vec{e}_\vartheta \times \vec{e}_r)$$
$$= + m r \sin\vartheta\, \vec{e}_\varphi$$

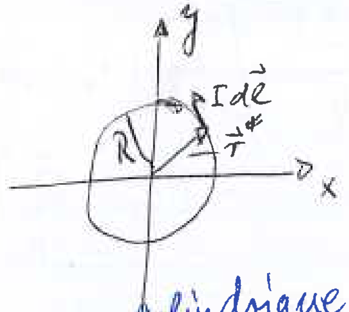
$$\Rightarrow \frac{\vec{m} \times \vec{r}}{r^3} = \frac{m \sin\vartheta\, \vec{e}_\varphi}{r^2}$$

seule composante de $\vec{A}$: $\vec{A} = A \vec{e}_\varphi = A_\varphi(r, \vartheta) \vec{e}_\varphi$

$$\Rightarrow \vec{\operatorname{rot}}\, \frac{\vec{m} \times \vec{r}}{r^3} = m\, \vec{\operatorname{rot}} \left(\frac{\sin\vartheta}{r^2} \vec{e}_\varphi \right) = \vec{e}_r \frac{1}{r \sin\vartheta} \left(\frac{\partial}{\partial \vartheta} \left(\sin\vartheta \frac{m \sin\vartheta}{r^2} \right) \right)$$
$$- \vec{e}_\vartheta \frac{1}{r} \frac{\partial}{\partial r} \left(r m \frac{\sin\vartheta}{r^2} \right)$$



spire dans le plan x, y:



choose:
origin at
center of loop:

en cylindrique:

$$d\vec\ell = R\, d\varphi\, \vec{e_\varphi}$$

$$\vec r = R\vec{e_\rho}$$

(par dév. multipolaire en magnétisme, hors de la spire):

generally:

$$\vec m = \frac{1}{2}\iiint_{V'} \vec r\,' \times \vec j(\vec r\,')\, dV'$$

$$\vec m = \frac{1}{2}\oint_C \vec r \times \left(I\, d\vec\ell\right) ;$$

$$= \frac{1}{2}\int_0^{2\pi} R^2 \vec{e_\rho} \times I \vec{e_\varphi}\, d\varphi = \frac{R^2 I}{\cancel{2}}\vec{e_z}\cdot \cancel{2}\pi$$

$$= \underbrace{\pi I R^2}_{m}\vec{e_z}$$

(Visualisez $\vec A_D(\vec r)$)

Calculer $\vec B(\vec r)$

$$\vec\nabla_r \times \vec A(\vec r) = \vec\nabla_r \times \frac{\mu_0}{4\pi}\frac{\vec m \times \vec r}{r^3}$$

$$= \frac{\mu_0}{4\cancel{\pi}}\vec\nabla_r \times \frac{\cancel{\pi} I R^2 \vec{e_z}\times\vec r}{r^3}$$

$$= \frac{\mu_0 I R^2}{4}\vec\nabla_r \times \left(\vec{e_z}\times\frac{\vec r}{r^3}\right)$$

$$\boxed{\vec{\mathrm{rot}}\,\left(\vec a(\vec r)\times\vec b(\vec r)\right) = \vec a(\vec\nabla\cdot\vec b) + (\vec b\cdot\vec\nabla)\vec a - (\vec a\cdot\vec\nabla)\vec b - \vec b(\vec\nabla\cdot\vec a)}$$

$$\Rightarrow \vec\nabla_r\times\vec A(\vec r) = \frac{\mu_0 I R^2}{4}\Big[\vec e_z\left(\vec\nabla_r\cdot\frac{\vec r}{r^3}\right) + \underbrace{\left(\frac{\vec r}{r^3}\cdot\vec\nabla\right)\vec e_z}_{=0} - (\vec e_z\cdot\vec\nabla_r)\frac{\vec r}{r_3} - \frac{\vec r}{r^3}\underbrace{(\vec\nabla_r\cdot\vec e_z)}_{=0}\Big]$$

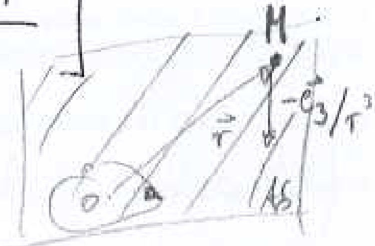
$$= \frac{\mu_0 I R^2}{4}\Big[\vec e_3\sum_{i=1}^{3}\vec e_i\frac{\partial}{\partial x_i}\cdot\sum_{j=1}^{3}\vec e_j\, x_j\Big(\sum_{k=1}^{3}x_k^2\Big)^{-3/2} - \frac{\partial}{\partial x_3}\frac{\vec r}{r^3}\Big]$$

$$= \frac{\mu_0 I R^2}{4}\Big[\vec e_3\sum_{i,j=1}^{3}\delta_{ij}\frac{\partial}{\partial x_i}x_j\Big(\sum_{k=1}^{3}x_k^2\Big)^{-\frac{3}{2}} - \frac{\vec e_3}{r^3} - \vec r\frac{\partial}{\partial x_3}\Big(\sum_{k=1}^{3}x_k^2\Big)^{-\frac{3}{2}}\Big]$$

$$= \frac{\mu_0 I R^2}{4}\Big[\frac{3\vec e_3}{r^3} + \vec e_3\sum_{i=1}^{3}x_i\Big(-\frac{3}{\cancel{2}}\cdot\cancel{2}x_i\Big)\frac{1}{r^5} - \frac{\vec e_3}{r^3} - \vec r\Big(-\frac{3}{\cancel{2}}\cdot\cancel{2}x_3\Big)\frac{1}{r^5}\Big]$$

$$= \frac{\mu_0 I R^2}{4}\Big[\frac{2\vec e_3}{r^3} - \frac{3\vec e_3}{r^5}r^2 + \frac{3x_3\vec r}{r^5}\Big]$$

$$\vec B(\vec r) = \boxed{\frac{\mu_0 I R^2}{4}\Big[\frac{3x_3\vec r}{r^5} - \frac{\vec e_3}{r^3}\Big]}$$



$\vec B$ toujours contenu dans plan AS pour I (spire) ✓

$$= \frac{\mu_0 I R^2}{4}\Big[\frac{3x_3^2\vec e_3}{r^5} + \frac{3x_3x_2\vec e_2}{r^5} + \frac{3x_3x_1\vec e_1}{r^5} - \frac{\vec e_3}{r^3}\Big]$$ ✓

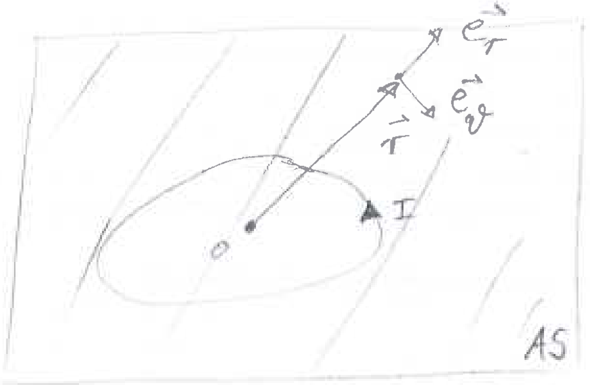
For points on the axis of the loop, we have

$$\vec B(x_3>R) = \frac{\mu_0 I R^2}{4}\Big[\frac{3x_3x_3\vec e_3}{x_3^5} - \frac{\vec e_3}{x_3^3}\Big] = \frac{\mu_0 I R^2}{\cancel{4}_2}\cdot\cancel{2}\,\frac{\vec e_3}{x_3^3}$$

$$= \vec{e}_r \frac{1}{r\cancel{\sin\vartheta}} \frac{m}{r^2} 2\cancel{\sin\vartheta}\cos\vartheta - \vec{e}_\vartheta \frac{m\sin\vartheta}{r} \left(-\frac{1}{r^2}\right)$$

$$= \left(\vec{e}_r \cdot 2\cos\vartheta + \vec{e}_\vartheta \sin\vartheta\right) \frac{m}{r^3}$$

$$\Rightarrow \vec{B} = \frac{\mu_0}{4\pi} \frac{m}{r^3} \left(2\cos\vartheta\, \vec{e}_r + \sin\vartheta\, \vec{e}_\vartheta\right)$$



$\vec{B}(\vec{r})$ est dans un plan AS contenant le point repéré par le vecteur position $\vec{r}$.