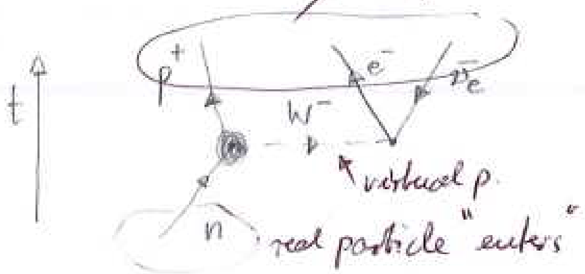
f: constante de désintégration : Γ "decay rate"



Feynman diagram for nuclear β^- decay.
(Involves Feynman calculus for two vertices : $\mathcal{M}$ scattering amplitude)

*(can be regarded at quark level, too.)

Γ : probability per time unit for a decay (of a particle)

$$|\mathcal{M}|^2 = \frac{d\sigma}{d\Omega}$$

$d\sigma$ ← section efficace (probabilité d'interaction entre particules) — absorption, diffusion, ou réaction

$d\Omega$ ← angle solide

généralement : $\mathcal{M} = \langle \psi_i | \hat{H}_{int} | \psi_f \rangle$

$\mathcal{M}$ via Feynman rules for QFT (taken from QED since fermions are all external particles):

$p_n, s_n = \frac{1}{2}$

$p_p, s_p = \frac{1}{2}$

$p_{e^-}, s_e = \frac{1}{2}$

$p_{\bar{\nu}_e}, s_\nu = \frac{1}{2}$

q_{W^-}

momenta

u_n inc. neutron

$\bar{u}_p$ outg. proton

$\bar{u}_e$ outg. electron

v_ν outg. antineutrino

factors (Dirac functions)

$$\boxed{-\frac{i g_w}{2\sqrt{2}} \gamma^\mu (1-\gamma^5) \quad (2\times)}$$

vertex factors

indicates parity violation!

$g_w = \sqrt{4\pi \alpha_w}$

$$\frac{-i\left(g_{\mu\nu} - q_\mu q_\nu / M_W^2 c^2\right)}{q^2 - M_W^2 c^2}$$ W^- propagator

$q^2 \ll M_{W^-}^2 c^2$ usually,

so $\approx \boxed{\dfrac{-i g_{\mu\nu}}{(M_{W^-} c)^2}}$

$$\boxed{\begin{array}{l}(2\pi)^4 \ \delta^4(p_n - p_p - q_W) \\ (2\pi)^4 \ \delta^4(-p_e - p_\nu - q_W)\end{array}}$$ conservation of energy + mom.

$$\boxed{\frac{i}{(2\pi)^4} \ d^4 q_W}$$

integration:

$$\left(\int -\frac{g_W^2}{8} \gamma^\mu (1-\gamma^5) \gamma^\nu (1-\gamma^5) \ \overbrace{\frac{-i g_{\mu\nu}}{M_W^2 c^2}}^{\approx} \ (2\pi)^8 \ \delta^4(p_n - p_p - q_W) \ \delta^4(-p_e - p_\nu + q_W) \ \frac{i}{(2\pi)^4} d^4 q_W \right)$$

$$\frac{-i \, i (2\pi)^8}{(2\pi)^4} \int -\frac{g_W^2}{8} \, u_n^{(s_n)}(p_n) \, \gamma^\mu (1-\gamma^5) \, \bar{u}_p^{(s_p)}(p_p) \, \frac{g_{\mu\nu}}{M_W^2 c^2} \, \bar{u}_e^{(s_e)}(p_e) \, \gamma^\nu (1-\gamma^5) \, u_\nu^{(s_\nu)}(p_\nu)$$

$$\cdot \, \delta^{(4)}(p_n - p_p - q_W) \, \delta^{(4)}(-p_e - p_\nu + q_W) \ d^4 q_W$$

$$i\mathcal{M} = -(2\pi)^4 \frac{g_W^2}{8 M_W^2 c^2} \, u_n^{(s_n)}(p_n) \, \gamma^\mu (1-\gamma^5) \, \bar{u}_p^{(s_p)}(p_p) \, g_{\mu\nu} \, \bar{u}_e^{(s_e)}(p_e) \, \gamma^\nu (1-\gamma^5) \, v_\nu^{(s_\nu)}(p_\nu) \, \underbrace{\delta^{(4)}(p_n - p_p - p_e - p_\nu)}_{\text{4-mom. conservation}}$$

... more theory, spins, helicity, Casimir's trick. → drop.

$\Rightarrow$

$$\sum_{\text{Spins}} |\mathcal{M}|^2 = 4 \left(\frac{g_W}{M_W c} \right)^4 (p_n \cdot p_\nu)(p_p \cdot p_e)$$

This can be evaluated in CM frame ($\sum_i \vec{p}_i = 0$)
⇒ expression with E included energy. (→ more details)

au repère au repos du neutron :

$$(p_n \cdot p_\nu) = \frac{E_n}{c}\frac{E_\nu}{c} - \underset{\vec{0}}{\vec{p}_n} \cdot \vec{p}_\nu \qquad E_n = \sqrt{m_n^2 c^4} = m_n c^2$$

$$= m_n E_\nu \qquad \left(E_\nu = \|\vec{p}_\nu\| c\right)$$

comme $p_n = p_p + p_e + p_\nu$ *

$$\Rightarrow (p_p + p_e)^2 = p_p^2 + p_e^2 + 2 p_p \cdot p_e$$

$$= \frac{E_p^2}{c^2} - \vec{p}_p^{\,2} + \frac{E_e^2}{c^2} - \vec{p}_e^{\,2} + 2 p_p \cdot p_e$$

$$= \frac{\vec{p}_p^{\,2} c^2 + m_p^2 c^4}{c^2} - \vec{p}_p^{\,2} + m_e^2 c^2 + 2 p_p \cdot p_e$$

$$= m_p^2 c^2 + m_e^2 c^2 + 2 p_p \cdot p_e$$

$$\overset{*}{=} (p_n - p_\nu)^2 = p_n^2 + p_\nu^2 - 2 p_n \cdot p_\nu$$

$$= \frac{E_n^2}{c^2} + \underbrace{\frac{E_\nu^2}{c^2} - \vec{p}_\nu^{\,2}}_{=0} - 2 p_n \cdot p_\nu$$

$$= m_n^2 c^2 - 2\, p_n \cdot p_\nu$$

**

$$- 2 m_n E_\nu$$

** $\Rightarrow$

$$p_p \cdot p_e = \frac{1}{2}\left(m_n^2 c^2 - m_p^2 c^2 - m_e^2 c^2 - 2 m_n E_\nu\right)$$

alors $$\sum_{\text{spins}} |\mathcal{M}|^2 = 4 \left(\frac{g_W}{M_W c}\right)^4 m_n E_\nu \left[\frac{1}{2}\left(m_n^2 c^2 - m_p^2 c^2 - m_e^2 c^2\right) - m_n E_\nu\right]$$

... travail! [Golden rule for decays]

$$\Rightarrow \Gamma = \frac{1}{4\pi^3 \hbar} \left(\frac{g_W}{2 M_W c^2}\right)^4 (m_e c^2)^5 \left[\frac{1}{15}(2a^4 - 9a^2 - 8)\sqrt{a^2-1} + a \ln\left(a + \sqrt{a^2-1}\right)\right]$$

avec $a = \dfrac{m_n - m_p}{m_e}$

donne $\tau = \dfrac{1}{\Gamma} = 1316$ [s] exp. : $\tau_{exp} = 898 \pm 16$ [s]

contribution élémentaire au taux de désintégration (ici) :

$$d\Gamma = \frac{\langle |\mathcal{M}|^2 \rangle}{2\hbar m_n} \left(\frac{c\, d^3\vec{p}_p}{(2\pi)^3\, 2E_p} \right) \left(\frac{c\, d^3\vec{p}_e}{(2\pi)^3\, 2E_e} \right) \left(\frac{c\, d^3\vec{p}_\nu}{(2\pi)^3\, 2E_\nu} \right) (2\pi)^4\, \delta^4(p_n - p_p - p_e - p_\nu)$$

avec $E_\nu = \|\vec{p}_\nu\| c$ $\qquad E_e = c\sqrt{\vec{p}_e^{\,2} + m_e^2 c^2}$ $\qquad E_p = c\sqrt{\vec{p}_p^{\,2} + m_p^2 c^2}$

intégration $d^3\vec{p}_p$:

d'abord : $\delta^4(p_n - p_p - p_e - p_\nu) = \delta\left(\frac{E_n}{c} - \frac{E_p}{c} - \frac{E_e}{c} - \frac{E_\nu}{c} \right) \delta^3(-\vec{p}_p - \vec{p}_e - \vec{p}_\nu)$

$$= \delta\left(m_n c - \sqrt{\vec{p}_p^{\,2} + m_p^2 c^2} - \sqrt{\vec{p}_e^{\,2} + m_e^2 c^2} - \|\vec{p}_\nu\| \right) \delta^3(-\vec{p}_p - \vec{p}_e - \vec{p}_\nu)$$

$$\Rightarrow d\Gamma = \frac{\langle |\mathcal{M}|^2 \rangle c^3}{16 (2\pi)^5 \hbar m_n} \, \frac{(d^3\vec{p}_e)(d^3\vec{p}_\nu) \quad \delta\left(m_n c - \sqrt{\|-\vec{p}_e - \vec{p}_\nu\|^2 + m_p^2 c^2} - \sqrt{\vec{p}_e^{\,2} + m_e^2 c^2} - \|\vec{p}_\nu\| \right)}{c\sqrt{\|-\vec{p}_e - \vec{p}_\nu\| + m_p^2 c^2} \; c\sqrt{\vec{p}_e^{\,2} + m_e^2 c^2} \; \|\vec{p}_\nu\| c}$$

intégration $d^3\vec{p}_\nu$:

d'abord : $d^3\vec{p}_\nu = \|\vec{p}_\nu\|^2 \, d\|\vec{p}_\nu\| \sin\vartheta \, d\vartheta \, d\varphi$ $\qquad$ en sphérique

$\left(r^2 \quad dr \cdot \sin\vartheta \, d\vartheta \, d\varphi \right.$ $\quad$ (analogue) $\Big)$