Chapter 3

Introduction to Nuclear Physics

This final chapter gives an introduction to some important aspects of nuclear physics. Various sources have been used, among them Griffith's book (see above), the monograph by Povh, Rith, Scholz, Zetsche, Rodejohann "Particles and Nuclei", and the interactive website `https://www.nndc.bnl.gov/nudat3/` of the Brookhaven National Laboratory that I highly recommend[1].

3.1 General Definitions

Let us begin with some terms and definitions. A **nuclide** (french: nucléide) is understood as a **bound state** composed of A nucleons (p^+, n) among which there are Z protons (p^+). This is a non-redundant and the standard definition, although sometimes the number of protons and neutrons is given. The information can be assembled into a **symbol for the nuclide**

$$^{\mathbf{A}}_{\mathbf{Z}}\mathbf{X}^{\mathbf{(Q)}}_{\mathbf{N}}$$

X denotes an element of the **periodic table of elements** (H, He, Li, ...)
N is the number of neutrons, $N = A - Z$

Sometimes, when the nuclide is understood to be an atomic nucleus

[1]The images in the manuscript have been created with the preceding version nudat2.

and electrons are present in the bound state, the total charge $Q = Z - \#e^-$ of the system can be given as well, depending on context. For example, the α particle is denoted as the nuclide ${}^4_2\mathrm{He}_2$. As an example where it is useful to add the total charges, reconsider the earlier weak decay of tritium which in terms of its fundamental process was given by Eq. (2.39). In **nuclear notation** at nucleon level this can now be written as

$$ {}^3_1\mathrm{H}_2 \longrightarrow {}^3_2\mathrm{He}^+_1 + e^- + \overline{\nu}_e \tag{3.1} $$

A neutron of tritium has decayed into a proton (which remains bound), and so A does not change. However, the proton number has changed and so we consider the product to be a nucleus of helium, not hydrogen. ${}^3_1\mathrm{H}_2$ is electrically neutral and has 1 electron. So does ${}^3_2\mathrm{He}^+_1$, but now we have 2 protons and so $Q = +1$ (denoted +). Note also the conservation laws, $L_e = L'_e = 1$ and $B = B' = 3$.

Further definitions:

- **Isotopes** are nuclides which have the **same proton number**, so $Z = Z'$. ${}^4_2\mathrm{He}$ and ${}^3_2\mathrm{He}$ are, therefore, isotopes. They always share the same element symbol.
- **Isotones** are nuclides which have the **same neutron number**, so $N = N' = A - Z = A' - Z'$. ${}^4_2\mathrm{He}_{N=2}$ and ${}^3_1\mathrm{H}_{N=2}$ are, therefore, isotones.
- **Isobars** are nuclides which have the **same nucleon number**, so $A = A'$. ${}^3_2\mathrm{He}$ and ${}^3_1\mathrm{H}$ are, therefore, isobars.

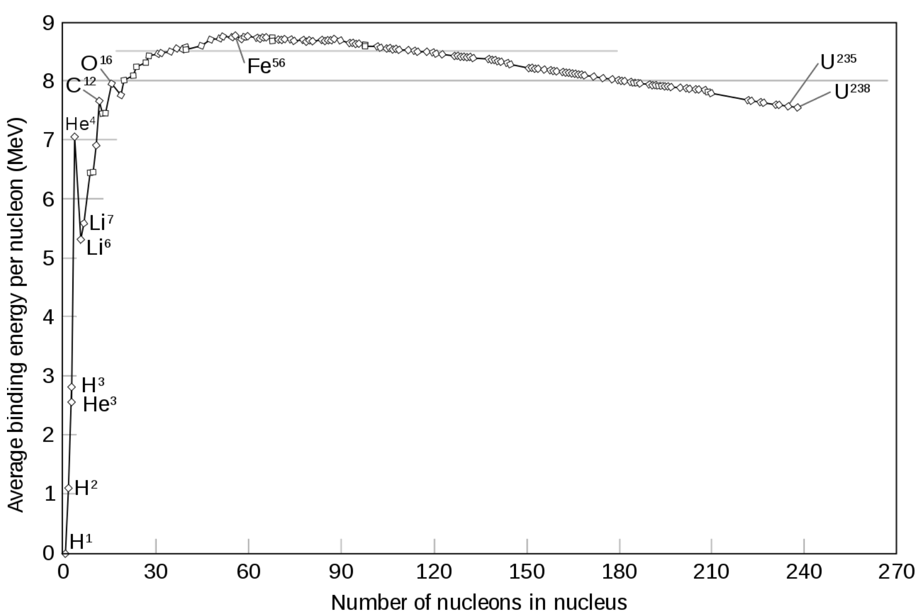
Another important quantity is the **nuclear binding energy**. It is defined as

$$ E_{\mathrm{bind}} = \left[Z m_{{}^1_1\mathrm{H}} + (A - Z) m_n - m_{{}^A_Z\mathrm{X}} \right] c^2 \tag{3.2} $$

where $m_{{}^1_1\mathrm{H}}$ is the rest mass of a hydrogen atom, m_n is the rest mass of a neutron, and $m_{{}^A_Z\mathrm{X}}$ is the rest mass of the neutral atom X under consideration[2].

Compare the nuclear binding energy Eq. (3.2) with the mass defect given in Eq. (1.159). If the mass defect is multiplied by c^2 then we can say that $m_{\text{sys}_{\text{before}}}c^2$ corresponds to the rest energy of the separated $Zm_{{}^1_1\text{H}}$ hydrogen atoms plus the rest energy of the separated $(A-Z)m_n$ neutrons, and $m_{\text{sys}_{\text{after}}}c^2$ corresponds to the rest energy of the bound nucleus composed of all these particles. In other words, the energy "lost", E'_{rad} in the formation of the nucleus **is** its binding energy.

E_{bind} is a positive quantity, $E_{\text{bind}} > 0$.

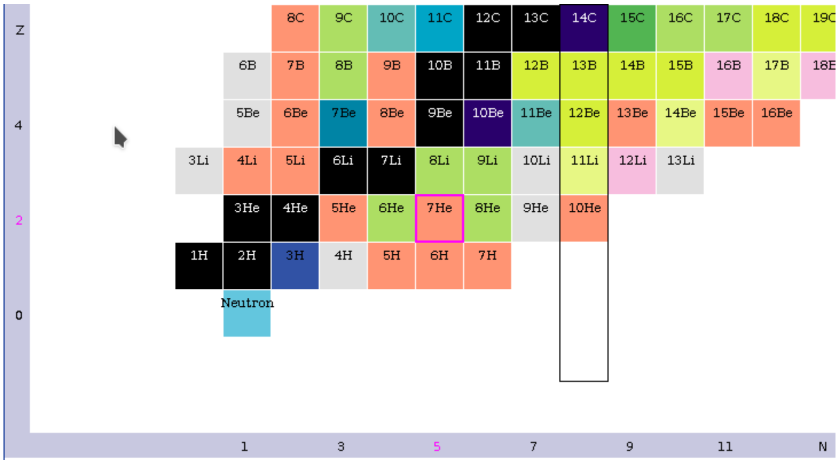


Nuclear binding energies per nucleon for common (abundant) isotopes of a given nuclide. The same trend as for the mass defect in Table 1.1 is observed.

3.2 Strong Isospin

A glance at a few of the simplest nuclides reveals an interesting fact. The following snapshot is from the aforementioned website at Brookhaven and shows the lower left corner of a (Z, N) diagram.

[2]Electron masses are generally included in this definition because neutral atoms are easier to "weigh" than ions and electrons are so light compared to nucleons.



Black nuclides are stable (very long lifetimes), colored nuclides decay in different ways (we will come back to that later). and an empty space means this nuclide has an unmeasurably short lifetime.

We see that ${}^2_1$H (p^+, n) is stable, but two protons or two neutrons do **not form a bound state**. Since there is no electrostatic repulsion between them it seemed particularly strange that two neutrons do not bind to each other. How can this be explained?

In a 1932 classic paper Werner Heisenberg came up with an explanation. It is based on the observation that the rest masses of the proton (938.3 $\left[\frac{\text{MeV}}{c^2}\right]$) and the neutron (939.6 $\left[\frac{\text{MeV}}{c^2}\right]$) are almost the same. So *via* Einstein's mass-energy equivalence, Eq. (1.138), the near-identity of their rest energies is a **near-degeneracy** of two different quantum-mechanical states.

Heisenberg proposed to write these two energies as E_{I,M_I} and $E_{I,M_I'}$ where I satisfies the algebra of an angular momentum and M_I is its projection onto the quantization axis. Now, we know the theorem that if $\left[\hat{H}, \hat{I}_k\right] = 0 \;\forall k \in \{1,2,3\}$ then $E_{I,M_I} = E_{I,M_I'}$. This in turn means that the Hamiltonian also commutes with a corresponding rotation, the generator of which is the operator $\hat{\mathbf{I}}$:

$$\left[\hat{H}, \hat{U}_I(\delta\varphi)\right] = 0 \tag{3.3}$$

where $\hat{U}_I(\delta\varphi) = e^{\frac{\imath}{\hbar}\delta\varphi\,\hat{\mathbf{e}}_n\cdot\hat{\mathbf{I}}}$.

The two nucleons can, therefore, be understood as two different quantum-mechanical microstates that are related by a rotation in a corresponding space. Since the number of microstates is two, Heisenberg conjectured that they form the **fundamental irreducible representation** of the Lie group $SU(2)$, called $\Gamma^{1/2}$, just like the spin of a fermion does. Since there is no additional angular momentum in the nucleon states but their degeneracy can be understood in terms of angular momentum algebra, I is called **isospin** (*like* spin), and $\hat{U}_I(\delta\varphi)$ is an infinitesimal rotation in the abstract **isospin space** which is the analog of spin space.

This has interesting consequences. First, if there is a symmetry (invariance of the Hamiltonian under isospin rotations) then there is a conservation law (*via* Noether's Theorem). The common force between an ensemble of protons and neutrons is the strong interaction, so this implies that

$$\left[\hat{H}_{\text{strong}}, \hat{U}_I(\delta\varphi)\right] = 0 \tag{3.4}$$

or in other words, the **strong interaction conserves isospin** *via* Heisenberg's equation of motion. This isospin is thus also called strong isospin.

This is a lot to swallow. But let's see if it helps us understand what is going on with bound states among protons and neutrons. Remember from the courses on symmetry that – without external fields – a spin 1/2 fermion has two degenerate states, $|m_s = 1/2\rangle$ and $|m_s = -1/2\rangle$ (that form the fundamental irreducible representation $\Gamma^{1/2}$). If we have two such particles, the spin eigenfunctions for the two-particle states represent a non-degenerate **singlet state** and a threefold-degenerate **triplet state**.

Now it is not hard to construct the isospin analog of the above finding. The general convention is to denote the proton and neutron as the following isospin states:

$$\left|p^{+}\right\rangle \equiv \left|I = \frac{1}{2}, M_I = \frac{1}{2}\right\rangle \tag{3.5}$$

$$\left|n\right\rangle \equiv \left|I = \frac{1}{2}, M_I = -\frac{1}{2}\right\rangle \tag{3.6}$$

By theoretical analogy, a system composed of **two** nucleons is then represented by one of the four possible isospin microstates:

$$\left|p^{+}\,p^{+}\right\rangle \equiv \left|I = 1, M_I = 1\right\rangle \tag{3.7}$$

$$\frac{1}{\sqrt{2}}\left[\left|p^{+}\,n\right\rangle + \left|n\,p^{+}\right\rangle\right] \equiv \left|I = 1, M_I = 0\right\rangle \tag{3.8}$$

$$\left|n\,n\right\rangle \equiv \left|I = 1, M_I = -1\right\rangle \tag{3.9}$$

This is the isospin triplet. Likewise,

$$\frac{1}{\sqrt{2}}\left[\left|p^{+}\,n\right\rangle - \left|n\,p^{+}\right\rangle\right] \equiv \left|I = 0, M_I = 0\right\rangle \tag{3.10}$$

which is the isospin singlet. From the conclusions on the theory of the Heisenberg Hamiltonian we know that if the interaction between the particles is repulsive (case of electrons and electromagnetism) then the spin triplet has lower energy. However, in the present case the respective interaction is **attractive** (strong interaction), and so we must conclude that here the **isospin singlet is more stable**[3]. And since the states $|p^{+}\,p^{+}\rangle$ and $|n\,n\rangle$ belong to the destabilized isospin triplet we have an explanation for the non-existence of such states!

Perhaps all of this seems like magic to you. After all, in order for the concept of strong isospin to be utterly convincing, it must manifest itself in more than just the proton-neutron states. Take a look again at the baryon octet diagram in the last chapter. The next four heavier

[3] The physical picture here is that the probability of finding two particles with the same isospin (projection) at the same point in space is zero, and so their mutual strong attraction is **reduced** on the average.

particles have rest masses

$$
\begin{aligned}
m_{\Sigma^+}(uus) &= 1189.4 \left[\frac{\text{MeV}}{c^2}\right] \\
m_{\Sigma^0}(uds) &= 1192.5 \left[\frac{\text{MeV}}{c^2}\right] \\
m_{\Sigma^-}(dds) &= 1197.3 \left[\frac{\text{MeV}}{c^2}\right]
\end{aligned}
$$

which can be arranged into an almost fully degenerate **isospin triplet** which transforms as to the 3-dimensional *irrep* $\Gamma^{I=1}$ of isospin $SU(2)$. The fourth particle is

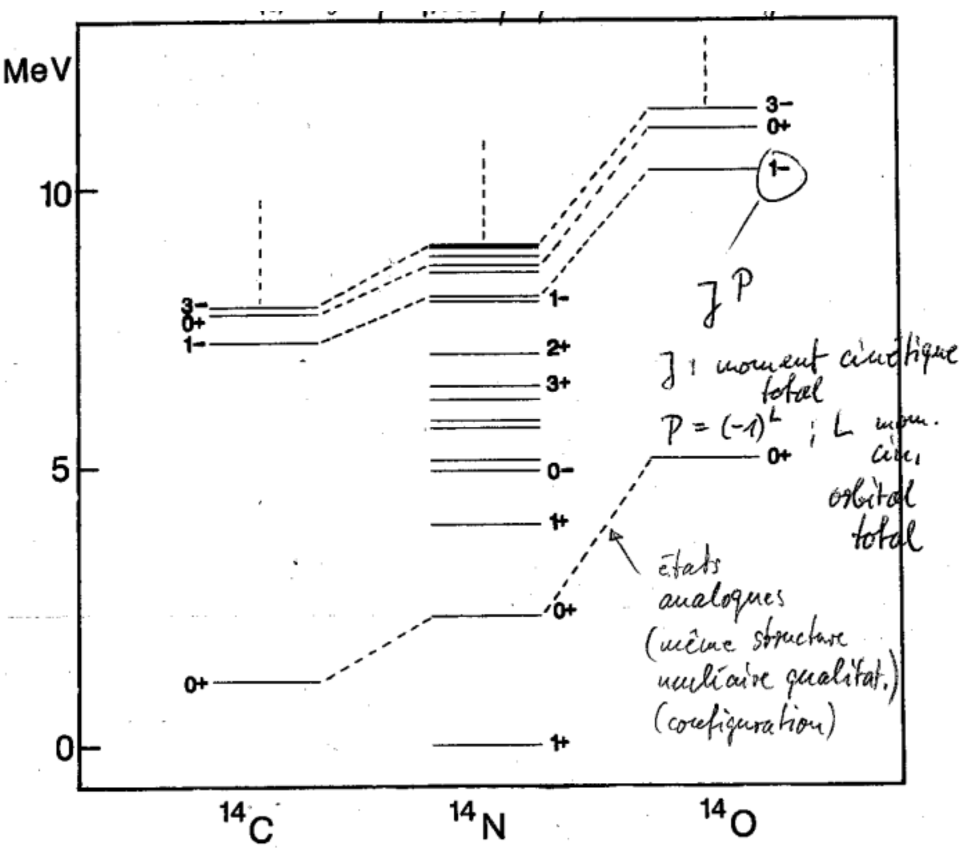
$$
m_{\Lambda}(uds) = 1115.6 \left[\frac{\text{MeV}}{c^2}\right]
$$

which forms an **isospin singlet** and transforms as to the 1-dimensional *irrep* $\Gamma^{I=0}$ of isospin $SU(2)$.

Then we have an isospin doublet for Θ^0 and Θ^-, and the concept carries on for the mesons where the π particles form an isospin triplet, the η a singlet, and so on[4].

Still not convinced? Take a look at the energetically lower part of the spectrum of the following three isobaric nuclei:

[4] Isospin is a **broken symmetry**. It is very weakly broken for the lighter particles and the breaking becomes greater for the heavier particles, essentially due to mass differences between the quarks of the Standard Model. Furthermore, it is a symmetry under strong interaction but not under electromagnetic or weak interaction.



Many-nucleon states are classified as to total angular momentum J and parity P. The nuclear ground state of $^{14}_{7}$N is aligned with $E = 0$.

Obviously, the spectra of $^{14}_{6}$C and $^{14}_{8}$O are very similar, but they differ qualitatively from the spectrum of $^{14}_{7}$N where many more states are observed at low energies and the nuclear ground state is different.

Now consider the isospin of these nuclides. The projection quantum numbers can be calculated because we know the number of protons and neutrons in either case[5]:

$$\begin{aligned}
M_I(^{14}_{6}\mathrm{C}) &= 6 \times (+\frac{1}{2}) + 8 \times (-\frac{1}{2}) = -1 \\
M_I(^{14}_{8}\mathrm{O}) &= 8 \times (+\frac{1}{2}) + 6 \times (-\frac{1}{2}) = +1 \\
M_I(^{14}_{7}\mathrm{N}) &= 7 \times (+\frac{1}{2}) + 7 \times (-\frac{1}{2}) = 0
\end{aligned}$$

Since $M_I(^{14}_{6}\mathrm{C}) = -1$ the lowest possible isospin quantum number for this nuclide is $I = 1$. This is because $M_I = 0$ does not exist here. So we can say that $I_{\min}(^{14}_{6}\mathrm{C}) = 1$. Likewise, $I_{\min}(^{14}_{8}\mathrm{O}) = 1$ but $I_{\min}(^{14}_{7}\mathrm{N}) = 0$

[5]Remember that for coupling angular momenta $\hat{\mathbf{I}} = \sum_j \hat{\mathbf{I}}(j)$ vectorially and so $M_I = \sum_j M_I(j)$.

because in the latter case $M_I = 0$ exists.

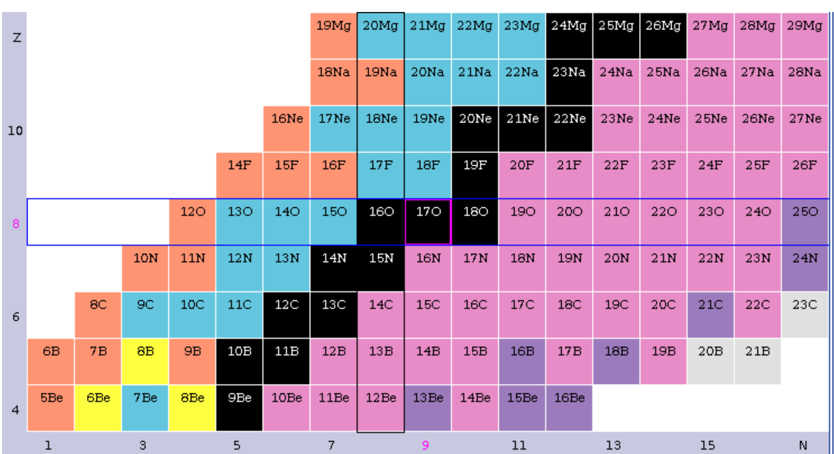
So we find that the nuclides differ qualitatively at the level of isospin. And since isospin is a symmetry of the strong interaction, the intrinsic interactions in the ${}^{14}_{7}\mathrm{N}$ nuclide differs from the other two nuclides[6].

In the bigger picture stable nuclei are therefore those where the number of protons equals the number of neutrons. For the lighter nuclei this general rule is very well fulfilled. As the nuclei become heavier, the mean electromagnetic Coulomb repulsion between the protons increases and gradually nuclei with growing neutron excess become the most stable isotopes.

3.3 Radioactive Decay

3.3.1 Decay Types

The following snapshot from the Brookhaven website is centered around a stable (black color) oxygen nuclide.



Nuclides around ${}^{17}_{8}\mathrm{O}$ display various ways of decaying, indicated by a color code.

All of the different decay modes are manifestations of **radioactive decay**. We will discuss them one by one.

■ Stable nuclide. Its **half-life**[7] is $> 10^{15}$ seconds.

[6] The exact reasons for the appearance of additional states is a matter of details.
[7] To be defined rigorously later.

As the neutron-to-proton ratio $\frac{N}{Z}$ increases (to the lower right) the nuclides become less stable. We are moving toward isospin states that are further removed from the stable isospin singlets (as discussed in the previous section). In this case the decay type is **beta (minus) decay** of one of the neutrons. In terms of the fundamental process it is denoted as

$$n \longrightarrow p^+ + e^- + \overline{\nu}_e \tag{3.11}$$

The β^- particle is synonymous with the electron. The proton typically remains bound in the nuclide.

If the neutron-to-proton ratio is increased even further, the nuclide decays by **neutron emission**. A neutron is ejected from the nuclide, written in nuclear notation:

$$^{A}_{Z}\mathrm{X}_N \longrightarrow {}^{A-1}_{Z}\mathrm{X}_{N-1} + {}^{1}_{0}\mathrm{n}_1 \tag{3.12}$$

The "new" nuclide $^{A-1}_{Z}\mathrm{X}_{N-1}$ is generally more stable than $^{A}_{Z}\mathrm{X}_N$, but it can further disintegrate *via* beta (minus) decay.

Unknown decay mode.

As the neutron-to-proton ratio $\frac{N}{Z}$ decreases (to the upper left) the nuclides decay *via* **beta (plus) decay** of one of the protons. As a fundamental process on its own, this would not happen because the rest mass of the neutron is greater than that of the proton (review the discussion in subsection 1.7.3). However, the nucleus can provide the required energy if the product nuclide has greater binding energy than the original nuclide. In nuclear notation $\beta^+(e^+)$ decay is written as

$$^{A}_{Z}\mathrm{X}_N \longrightarrow {}^{A}_{Z-1}\mathrm{Y}_{N+1} + e^+ + \nu_e \tag{3.13}$$

The underlying fundamental process is

$$p^+ \longrightarrow n + e^+ + \nu_e \tag{3.14}$$

If electrons are present (such as in an atom), a competing process can occur which is called **electron capture**:

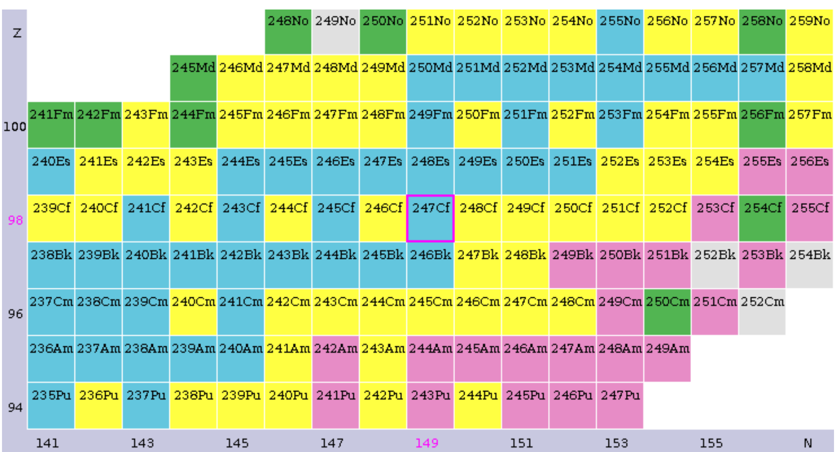
$$p^+ + e^- \longrightarrow n + \nu_e \tag{3.15}$$

Note that electron capture is a crossed reaction of the fundamental process underlying $\beta^+(e^+)$ decay.

As the neutron-to-proton ratio decreases even more, a proton is ejected from the unstable nucleus, according to

$${}^{A}_{Z}\mathrm{X}_N \longrightarrow {}^{A-1}_{Z-1}\mathrm{Y}_N + {}^{1}_{1}\mathrm{H}^+_0 \tag{3.16}$$

There are two more types of nuclear decay that are only observed for heavy nuclei:



For nuclides with $Z > 50$ and a relatively small neutron-to-proton ratio **α-decay** is observed[7]:

$${}^{A}_{Z}\mathrm{X}_N \longrightarrow {}^{A-4}_{Z-2}\mathrm{Y}_{N-2} + {}^{4}_{2}\mathrm{He}^{2+}_2 \tag{3.17}$$

For nuclides with $Z > 82$ **spontaneous fission** may occur:

$${}^{A}_{Z}\mathrm{X}_N \longrightarrow {}^{A'}_{Z'}\mathrm{Y}_{A'-Z'} + {}^{A-A'}_{Z-Z'}\mathrm{W}_{A-Z-(A'-Z')} \tag{3.18}$$

Here's an example of spontaneous fission of Californium (Cf):

$${}^{252}_{98}\mathrm{Cf}_{154} \longrightarrow {}^{140}_{54}\mathrm{Xe}_{86} + {}^{108}_{44}\mathrm{Ru}_{64} + 4\,{}^{1}_{0}\mathrm{n}_1 \tag{3.19}$$

In this case the fission into Xenon and Ruthenium is accompanied by the emission of four neutrons.

[7]This criterion is confirmed in the region where stable nuclides still exist. In the above section, however, there are no more stable structures among the nuclides of Bk (Berkelium), Es (Einsteinium), Fm (Fermium), etc.

3.3.2 Half Life

An important quantity for classifying the stability of nuclides is their **half life**, $t_{1/2}$. The half life is the instant in time when half of an ensemble N of particles (assumed to exist at $t = 0$) is still there. It can be determined analytically if the so-called **decay rate** Γ is known. The decay rate is the **probability of decay per time unit**[9].

Be $N(t)$ the number of particles at instant t and suppose that $\Gamma(\text{particle}) > 0$ is known. We define

$$\Delta N(t) := N(t_f) - N(t_i) < 0 \quad ; \quad t_f > t_i \tag{3.20}$$

the change in number of particles which is linearly[10] proportional to the number of existing particles at t, the respective time interval Δt, and the decay rate. We can write

$$\Delta N(t) = -\Gamma\, N(t)\, \Delta t \tag{3.21}$$

or in differential form

$$\begin{aligned} \frac{dN(t)}{dt} &= -\Gamma\, N(t) \\ \dot{N} &= -\Gamma\, N(t) \end{aligned}$$

The general solution of this linear homogeneous first-order differential equation is

$$N(t) = N(t=0)\, e^{\int_0^t -\Gamma\, dt'} = N(t=0)\, e^{-\Gamma\, t} \tag{3.22}$$

The number of existing particles at instant $t_{1/2}$ is, by definition,

$$N(t_{1/2}) = \frac{N(t=0)}{2} \tag{3.23}$$

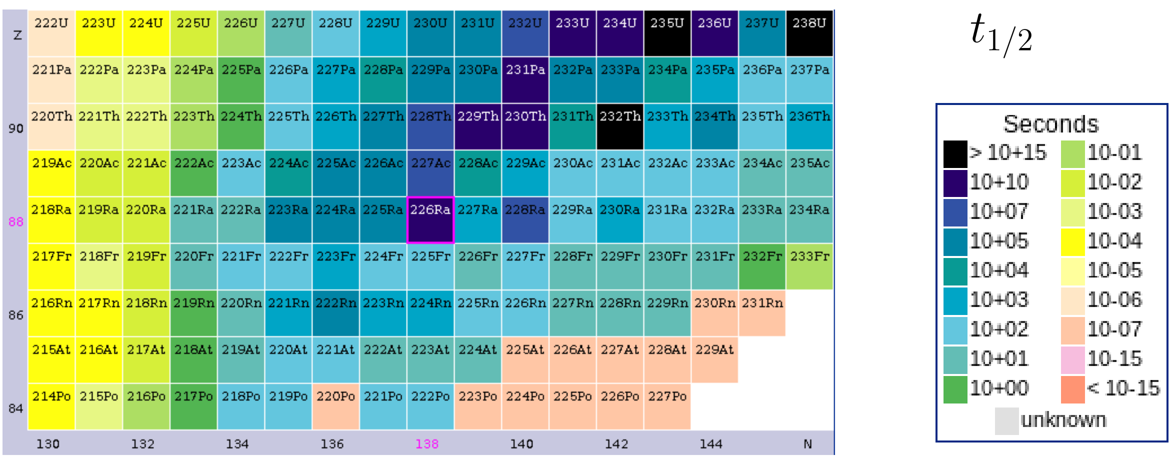
[9] In theory, this is the hard part. Γ can be calculated in the framework of Quantum Field Theory (QFT) invoking Feynman's calculus. If you are seriously interested, Griffith's book on elementary particles explains how to do it, in the final chapters. It does *not* explain the formal background of QFT, but how you get decay rates from Feynman rules. This is by itself not an easy exercise.

[10] The assumption is made that no decay affects any other.

Using this in Eq. (3.22) results in

$$
\begin{aligned}
N(t_{1/2}) = \frac{N(t=0)}{2} &= N(t=0)\, e^{-\Gamma\, t_{1/2}} \\
\Leftrightarrow -\Gamma\, t_{1/2} &= \ln\left(\frac{1}{2}\right) = -\ln\left(\frac{2}{1}\right) = -\ln 2 \\
t_{1/2} &= \frac{\ln 2}{\Gamma}
\end{aligned}
$$

So once the decay rate is known the half life is easy to calculate. The following chart shows a section of the nuclide table with associated half lives:



Thorium ${}^{232}_{90}\mathrm{Th}_{142}$ and Uranium ${}^{235}_{92}\mathrm{U}_{143}$ are stable nuclides with $t_{1/2} > 10^{15}$ [s]. The latter is used in *induced* nuclear fission where it absorbs a neutron to briefly form ${}^{236}_{92}\mathrm{U}_{144}$. This nuclide has a long half life of $\approx 10^{10}$ [s], but it is produced in an excited state which rapidly undergoes fission and releases excess energy.

3.4 Nuclear Structure - Nuclear Shell Model