E_0 : tête dipôle arrive en O : coïncidence.

$$(c\overset{=0}{t_0}, 0)_K \qquad (c\overset{=0}{t_0'}, 0)_{K'}$$

E_1 : queue du dipôle arrive en O :

$$(ct_1, ~~d_x~~ 0)_K \qquad (ct_1', -d_x')_{K'}$$

$$\Rightarrow \begin{pmatrix} ct_1 \\ 0 \end{pmatrix} = \begin{pmatrix} \gamma & \frac{v}{c}\gamma \\ \frac{v}{c}\gamma & \gamma \end{pmatrix} \begin{pmatrix} ct_1' \\ -d_x' \end{pmatrix}$$

$$ct_1 = \gamma c t_1' - \frac{v}{c}\gamma d_x'$$

$$0 = v\gamma t_1' - \gamma d_x' \quad \Leftrightarrow \quad t_1' = \frac{1}{v} d_x'$$

$$\Rightarrow ct_1 = \frac{\gamma c}{v} d_x' - \frac{v}{c}\gamma d_x' = d_x'\gamma\left(\frac{c}{v} - \frac{v}{c}\right)$$

$$t_1 = d_x'\gamma\left(\frac{1}{v} - \frac{v}{c^2}\right)$$

relier longueur propre en K à t_1 :

$$v = \frac{d_x}{t_1} \quad \Leftrightarrow \quad t_1 = \frac{d_x}{v}$$

$$\Rightarrow \frac{d_x}{v} = d_x'\gamma\left(\frac{1}{v} - \frac{v}{c^2}\right)$$

$$d_x = d_x'\gamma\left(1 - \frac{v^2}{c^2}\right) = d_x'\gamma\frac{1}{\gamma^2} = \frac{d_x'}{\gamma}$$

$$\frac{1}{\gamma} \overset{?}{=} \left(1 - \frac{\gamma}{1+\gamma}\frac{v^2}{c^2}\right)$$

$$1 = \gamma - \frac{\gamma^2}{1+\gamma}\frac{v^2}{c^2} = \gamma - \frac{\frac{v^2}{c^2\cancel{(1}-\frac{v^2}{\cancel{c^2}}}}{1+\gamma}$$

$$= \gamma - \frac{\frac{v^2}{c^2-v^2}}{1+\gamma} = \frac{\gamma+\gamma^2-\frac{v^2}{c^2-v^2}}{1+\gamma} = \frac{\gamma+\frac{c^2}{c^2-v^2}-\frac{v^2}{c^2-v^2}}{1+\gamma}$$

$$= \frac{\gamma+1}{\gamma+1} \quad ✓$$

$\Rightarrow$ general movement :

$$\vec{d}_f(K) = \vec{d}_f(K') - \frac{\gamma}{1+\gamma}\underbrace{\frac{\vec{v}}{c}}_{\frac{\text{vitesse relative}}{c}}\Big(\underbrace{\vec{d}_e(K')\cdot\frac{\vec{v}}{c}}_{\text{projection en dir. de vitesse relative}}\Big)$$

$$\boxed{7p \quad (+1p)}$$

$$\left\{b^{\mu}_{\text{réelle}}\right\} \doteq \begin{pmatrix} \frac{\gamma^4}{c} v a \\ \frac{\gamma^4}{c^2} v^2 a + \gamma^2 a \end{pmatrix} \qquad 1$$

$$b_{r.}^2 = \frac{\gamma^8}{c^2} v^2 a^2 - \frac{\gamma^8}{c^4} v^4 a^2 - \frac{2\gamma^6}{c^2} v^2 a^2 - \gamma^4 a^2$$

$$= \left(1 - \frac{v^2}{c^2}\right) \frac{\gamma^8}{c^2} v^2 a^2 - \frac{2\gamma^6}{c^2} v^2 a^2 - \gamma^4 a^2$$

$$= \frac{1}{\cancel{\gamma^2}} \frac{\gamma^{\cancel{8}6}}{c^2} v^2 a^2 - \frac{2\gamma^6}{c^2} v^2 a^2 - \gamma^4 a^2$$

$$= -\frac{\gamma^6}{c^2} v^2 a^2 - \gamma^4 a^2$$

$$= -\frac{1}{1 - \frac{v^2}{c^2}} \frac{v^2}{c^2} \gamma^4 a^2 - \gamma^4 a^2$$

$$= \left(-\frac{v^2}{c^2 - v^2} - 1\right) \gamma^4 a^2$$

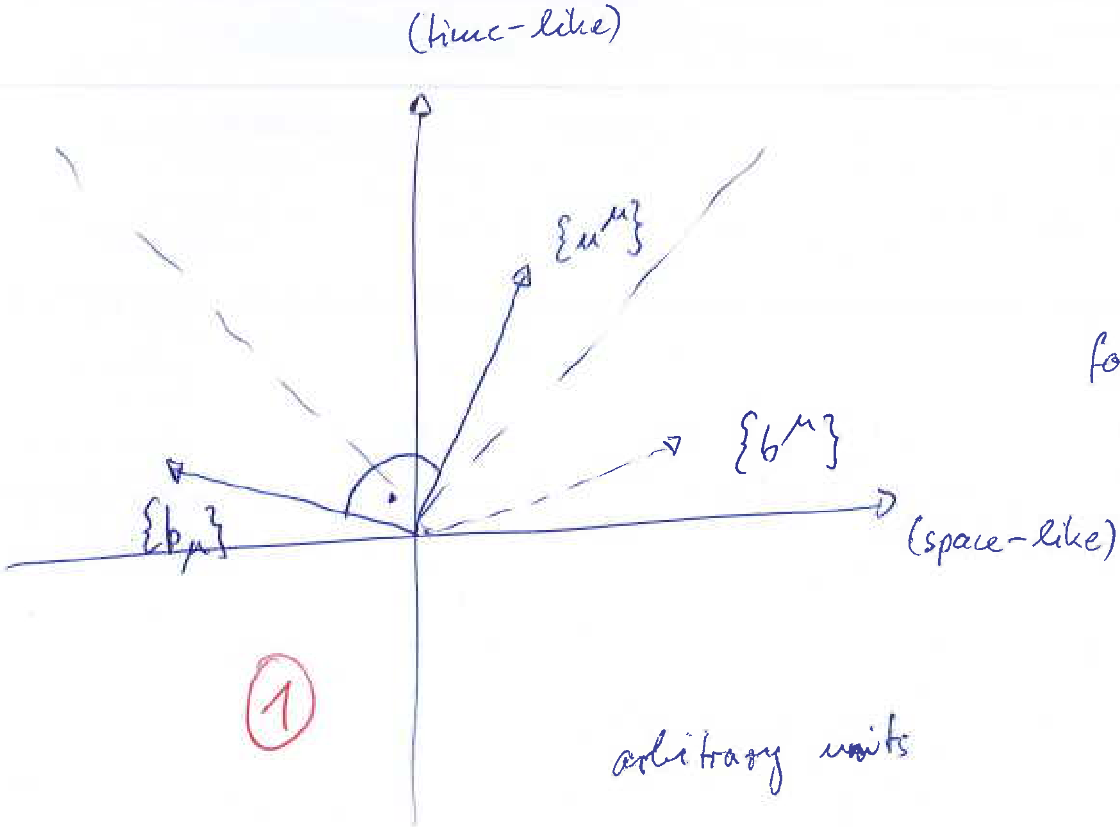
$$= \frac{\cancel{-v^2} - c^2 \cancel{+v^2}}{c^2 - v^2} \gamma^4 a^2$$

$$= -\frac{c^2}{\underbrace{c^2 - v^2}_{>0}} \gamma^4 a^2 \quad 2 \qquad < 0$$

$\Rightarrow$ b de genre espace. 1

relation avec u : $b^{\mu} u_{\mu} = 0 = b_{\mu} u^{\mu}$ 1

$\Rightarrow$ u de genre temps. 1



for $a>0$

$\boxed{7p}$

3. $A^\mu \to A^\mu + \partial^\mu \Lambda(x)$

partie "spatiale" : $\partial^\mu = -\partial_\mu = -\frac{\partial}{\partial x^\mu}$ pour $\mu \in \{1,2,3\}$ 1

$\vec{A} \to \vec{A} - \vec{\nabla}\Lambda(x)$ 1

$\vec{B} = \vec{\nabla} \times \vec{A}$

$\Rightarrow \vec{B} = \vec{\nabla}\times\vec{A} - \underbrace{\vec{\nabla}\times\vec{\nabla}\Lambda(x)}_{=\vec{0}} = \vec{B}$ invariant. 1

$\vec{E} = -\vec{\nabla}V - \frac{1}{c}\frac{\partial}{\partial t}\vec{A}$

$\Rightarrow \vec{E} = -\vec{\nabla}V - \frac{1}{c}\frac{\partial}{\partial t}\vec{A} + \frac{1}{c}\frac{\partial}{\partial t}\vec{\nabla}\Lambda(x)$ 1

$= -\vec{\nabla}\left(V - \frac{1}{c}\frac{\partial}{\partial t}\Lambda(x)\right) - \frac{1}{c}\frac{\partial}{\partial t}\vec{A}$

$\Rightarrow \vec{E}$ invariant si 1

$V \to V + \frac{1}{c}\frac{\partial}{\partial t}\Lambda(x) = V + \frac{\partial}{\partial x_0}\Lambda(x) = A^0 + \partial^0\Lambda(x)$ 1

cela est juste la composante de genre temps de l'éq. de transformation.

□.

$\boxed{6\text{p}}$