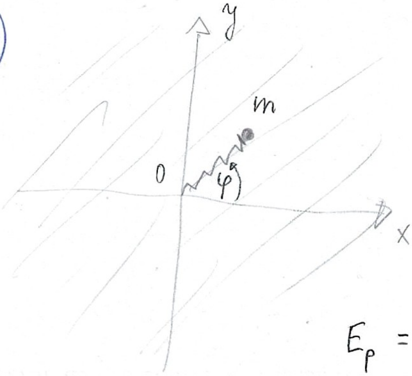
⑦



$\vec{F} = -k\,\vec{OM}$

en polaire :

$\vec{OM} = \rho\,\vec{e_\rho}$

$\vec{F} = -k\rho\,\vec{e_\rho}$

1.

$$E_p = -\int \vec{F}\cdot d\rho\,\vec{e_\rho} = +\frac{k}{2}\rho^2$$

teste : $-\vec{\nabla} E_p = -\vec{e_\rho}\frac{\partial}{\partial\rho}\left(+\frac{k}{2}\rho^2\right) = -k\rho\,\vec{e_\rho} = \vec{F}$ ✓

(Gradient en coordonnées polaires :

$$\vec{\nabla} = \vec{e_\rho}\frac{\partial}{\partial\rho} + \frac{1}{\rho}\vec{e_\varphi}\frac{\partial}{\partial\varphi}$$)

2. $E_m = E_c + E_p$

$$E_c = \frac{m}{2}\vec{v}^{\,2} \qquad \vec{v} = \dot\rho\,\vec{e_\rho} + \rho\dot\varphi\,\vec{e_\varphi}$$

$$\Rightarrow \vec{v}^{\,2} = \dot\rho^2 + \rho^2\dot\varphi^2$$

$$\Rightarrow E_c = \frac{m}{2}\left(\dot\rho^2 + \rho^2\dot\varphi^2\right)$$

$$\Rightarrow E_m = \frac{m}{2}\dot\rho^2 + \frac{m}{2}\rho^2\dot\varphi^2 + \frac{k}{2}\rho^2$$

$$\left[\frac{d}{dt}E_m = m\dot\rho\ddot\rho + \frac{m}{2}\left(2\rho\dot\rho\dot\varphi^2 + 2\rho^2\dot\varphi\ddot\varphi\right) + \frac{k}{2}2\rho\dot\rho\right]$$

pas de frottement, gravitation compensée par coussin d'air, seule force est ressort, conservative (dérive d'un potentiel)

$\Rightarrow E_m$ conservée.

3. $\vec{L}_0 = \vec{OM} \times m\vec{v}$

$$= m\rho\,\vec{e_\rho} \times \dot\rho\,\vec{e_\rho} + \rho\dot\varphi\,\vec{e_\varphi}$$

$$\vec{L}_0 = m\rho^2\dot\varphi\,\vec{e_z}$$

$$\vec{M}_O = \vec{OM} \times \vec{F}$$
$$= \rho \vec{e_\rho} \times (-k\rho \vec{e_\rho}) = \vec{0}$$

$$\Rightarrow \frac{d\vec{L}_O}{dt} = 0 \quad \Rightarrow \quad \frac{dL_z}{dt} = 0 \iff \frac{d}{dt} m\rho^2 \dot{\varphi} = 0$$

5. $$E_m = \frac{m}{2}\dot{\rho}^2 + E_{p\,eff} = \frac{m}{2}\dot{\rho}^2 + \frac{m}{2}\rho^2\dot{\varphi}^2 + \frac{k}{2}\rho^2$$

$$\Rightarrow E_{p\,eff} = \frac{m}{2}\rho^{2}\dot{\varphi}^2 + \frac{k}{2}\rho^2$$

$$\|\vec{L}\| = m\rho^2\dot{\varphi}$$

$$\Rightarrow E_{p\,eff} = 2\|\vec{L}\|\dot{\varphi} + \frac{k}{2}\rho^2$$