1.3 Produit mixte

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \cdot \begin{pmatrix} b_2 c_3 - b_3 c_2 \\ b_3 c_1 - b_1 c_3 \\ b_1 c_2 - b_2 c_1 \end{pmatrix}$$

$$= a_1 b_2 c_3 - a_1 b_3 c_2 + a_2 b_3 c_1 - a_2 b_1 c_3 + a_3 b_1 c_2 - a_3 b_2 c_1$$

$$= (a_2 b_3 - a_3 b_2) c_1 + (a_3 b_1 - a_1 b_3) c_2 + (a_1 b_2 - a_2 b_1) c_3$$

$$= \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix} \cdot \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = (\vec{a} \times \vec{b}) \cdot \vec{c} = \vec{c} \cdot (\vec{a} \times \vec{b})$$

on peut continuer pour montrer

$$= (\vec{c} \times \vec{a}) \cdot \vec{b}$$

$$= (c_2 a_3 - c_3 a_2) b_1 + (c_3 a_1 - c_1 a_3) b_2 + (c_1 a_2 - c_2 a_1) b_3$$

$$= (\vec{c} \times \vec{a}) \cdot \vec{b} = \vec{b} \cdot (\vec{c} \times \vec{a})$$

donc

permutations circulaires possibles.

1.4 Règle "bac – cab"

$$\vec{a}\times(\vec{b}\times\vec{c}) = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \times \begin{pmatrix} b_2c_3 - b_3c_2 \\ b_3c_1 - b_1c_3 \\ b_1c_2 - b_2c_1 \end{pmatrix} = \begin{pmatrix} a_2b_1c_2 - a_2b_2c_1 - a_3b_3c_1 + a_3b_1c_3 \\ a_3b_2c_3 - a_3b_3c_2 - a_1b_1c_2 + a_1b_2c_1 \\ a_1b_3c_1 - a_1b_1c_3 - a_2b_2c_3 + a_2b_3c_2 \end{pmatrix}$$

$$\vec{b}(\vec{a}\cdot\vec{c}) - \vec{c}(\vec{a}\cdot\vec{b}) = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}\left[a_1c_1 + a_2c_2 + a_3c_3\right] - \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}\left[a_1b_1 + a_2b_2 + a_3b_3\right]$$

$$= \begin{pmatrix} \cancel{b_1a_1c_1} + b_1a_2c_2 + b_1a_3c_3 - \cancel{b_1a_1c_1} - b_2a_2c_1 - b_3a_3c_1 \\ b_2a_1c_1 + \cancel{b_2a_2c_2} + b_2a_3c_3 - b_1a_1c_2 - \cancel{b_2a_2c_2} - b_3a_3c_2 \\ b_3a_1c_1 + b_3a_2c_2 + \cancel{b_3a_3c_3} - a_1b_1c_3 - a_2b_2c_3 - \cancel{a_3b_3c_3} \end{pmatrix} \checkmark$$