6 Modèle de champ pour noyaux légers

$$\rho = \rho_0 \left(1 - \frac{r^2}{a^2}\right) \quad , \quad r \leq a$$

$$\rho = 0 \quad , \quad r > a$$

Q1 charge totale

r : coordonnée radiale, donc problème sphérique

$$Q = \iiint_{\mathcal{V}} \rho(r)\, d\mathcal{V} = \int_0^a \rho(r)\, r^2\, dr \int_0^{\pi} \sin\vartheta\, d\vartheta \int_0^{2\pi} d\varphi$$

$$= \int_0^a \left[\rho_0 r^2 - \frac{\rho_0}{a^2} r^4\right] dr \cdot 2 \cdot 2\pi$$

$$= 4\pi \left[\frac{1}{3}\rho_0 r^3 - \frac{\rho_0}{5a^2} r^5\right]_0^a$$

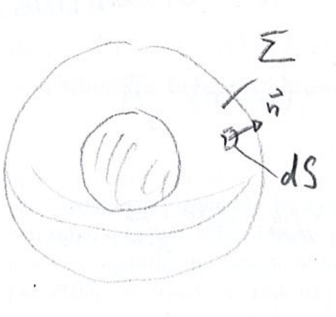
$$= 4\pi \left[\frac{\rho_0}{3} a^3 - \frac{\rho_0}{5} a^3\right]$$

$$Q = \frac{8\pi}{15} \rho_0\, a^3$$

$$\dim(\rho_0) = \frac{Q}{\mathcal{V}}$$

Q2 Champ électrostatique

surface de Gauss sphérique



sym. $\Rightarrow \vec{E} = E(r)\, \vec{e_r}$

$$\oiint_{\Sigma} \vec{E} \cdot \vec{n}\, dS = \int_0^{\pi}\int_0^{2\pi} E(r)\, \vec{e_r} \cdot \vec{e_r}\, r^2 \sin\vartheta\, d\vartheta\, d\varphi$$

$$= 4\pi r^2 E(r)$$

1) $r > a$ (extérieur)

$$Q_{in} = \frac{8\pi}{15} \rho_0 a^3$$

avec T.d.G. : $\cancel{4\pi} r^2 E(r) = \frac{\overset{2}{\cancel{8\pi}}}{15} \frac{\rho_0 a^3}{\varepsilon_0}$

$$E(r) = \frac{2 \rho_0 a^3}{15 \varepsilon_0} \frac{1}{r^2}$$

$$\boxed{\vec{E}_e(r) = \frac{2 \rho_0 a^3}{15 \varepsilon_0} \frac{1}{r^2} \vec{e}_r}$$

2) $r \leq a$ (intérieur)

$$Q_{in} = \int_0^r \rho(r') r'^2 dr' \quad \Omega$$

$$= 4\pi \left[\frac{\rho_0}{3} r^3 - \frac{\rho_0}{5a^2} r^5 \right]$$

T.d.G. $\Rightarrow$ $E(r) = \frac{\rho_0}{3\varepsilon_0} r - \frac{\rho_0}{5a^2 \varepsilon_0} r^3$

$$\boxed{\vec{E}_i(r) = \left[\frac{\rho_0}{3\varepsilon_0} r - \frac{\rho_0}{5a^2 \varepsilon_0} r^3 \right] \vec{e}_r}$$

continuité ?

$$E_e(r=a) = \frac{2 \rho_0 a}{15 \varepsilon_0}$$

$$E_i(r=a) = \frac{\rho_0 a}{3 \varepsilon_0} - \frac{\rho_0 a}{5 \varepsilon_0} = \frac{2 \rho_0 a}{15 \varepsilon_0} \quad \checkmark$$

Q3 : Potentiel électrostatique

$$\int_{M_i}^{M_f} \vec{E} \cdot d\vec{l} = V(M_i) - V(M_f)$$

on choisit $M_f = \{M_i, M_e\}$

$M_i = \infty$

1) Potentiel à $r > a$

$$\int_{\infty}^{r(M_e)} \frac{2\rho_0 a^3}{15\varepsilon_0} \frac{1}{r'^2} \underbrace{\vec{e}_{r'} \cdot \vec{e}_{r'}}_{=1} \, dr' = \underbrace{V(\infty)}_{!=0} - V(r)$$

$$-\frac{2\rho_0 a^3}{15\varepsilon_0} \, r'^{-1} \Big|_{\infty}^{r} = -V(r)$$

$$\boxed{V_e(r) = \frac{2\rho_0 a^3}{15\varepsilon_0} \frac{1}{r}} - 0$$

2) Potentiel à $r \leq a$

$$V_i = V_e(r=a) - \int_{r=a}^{r} \vec{E}_i \cdot d\vec{l}$$

$$= \frac{2\rho_0 a^2}{15\varepsilon_0} - \int_a^r \left[\frac{\rho_0}{3\varepsilon_0} r' - \frac{\rho_0}{5a^2\varepsilon_0} r'^3 \right] dr'$$

$$= \frac{2\rho_0 a^2}{15\varepsilon_0} - \frac{\rho_0}{6\varepsilon_0} r'^2 \Big|_a^r + \frac{\rho_0}{20a^2\varepsilon_0} r'^4 \Big|_a^r$$

$$= \frac{2\rho_0 a^2}{15\varepsilon_0} - \frac{\rho_0}{6\varepsilon_0} r^2 + \frac{\rho_0 a^2}{6\varepsilon_0} + \frac{\rho_0}{20a^2\varepsilon_0} r^4 - \frac{\rho_0 a^2}{20\varepsilon_0}$$

$$\boxed{V_i(r) = \frac{8 + \cancel{10} - 3}{\cancel{60}} \frac{1}{4} \frac{\rho_0 a^2}{\varepsilon_0} - \frac{\rho_0}{6\varepsilon_0} r^2 + \frac{\rho_0}{20a^2\varepsilon_0} r^4}$$